Making Math a Definition of the Situation: Families as Sites for Mathematical Practices

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We present three cases showing families’ competence in mathematical problem solving as a practical aspect of daily life. At home, parents and children engaged creatively in solving math-relevant problems. They used a combination of everyday practices and school forms, but generally did not recognize mathematics in their problem solving. The findings invite new forms of participation that bring families into discussions of math-relevant situations and relates them to their children’s school math.

Families are recognized as the first and foremost learning settings for children. It is widely believed that family involvement drives school achievement, yet parents tend to leave math to the school as their children rise through the grades and move beyond arithmetic. As we show in this article, this perceived alienation to school math need not be a given. This research, which was part of a project at the intersection of critical design ethnography and action research, reveals how family activities are rich in discourse and practices that actually instantiate a great many math relevant practices. We learned daily life provides many contexts for mathematical problem solving, that parents generally grapple with the problems they encounter productively and often to their satisfaction, and on some occasions, invoke the frames, methods, and algorithms of school math—all while rarely identifying what they do as “math.” Our descriptions of families engaged mathematically suggest connections and alignments with the math done at school. This is important because learning about and acknowledging the kinds of problem solving that families do can open up opportunities for parents to interact productively with their children’s mathematics learning.

We centered our inquiry on three intersecting questions: (1) What is the math people do in the course of family life and how can it be characterized? (2) How do parents of middle school students participate in their children’s schools, and what relevance does that have to their children’s school math careers? (3) Can what we learn offer any suggestions for improved math learning or interactions among the school and the home? The research examines how people engaged in mathematically relevant ways as they moved through their everyday activities and the ways they talked about and gave meaning to those experiences. It considers the social and cultural contexts within which people participated, with focus on the contexts that gave rise to mathematical practices and school engagements. We consider how math at home was socially constructed, situationally defined, and cognitively activated (Chaiklin and Lave 1993; Cole 1996; Lave 1988; Vygotsky 1978). When we distinguish between school math and family math, we are speaking of a difference in problem contexts and participatory structures (National Council of Teachers of Mathematics [NCTM] 2000). The content and topics are similar, but the ways in which people participate in problem solving in classrooms versus everyday life are quite different. As we show in this article, how problems are introduced, techniques for problem solving, evaluation, and social arrangements are all distinct. Although this article analyzes daily life problem solving and does not analyze school math problem solving, we show that family mathematics is often based on problems that present themselves in daily circumstances, that are often solved with attention to social relationships.
rather than individual achievement, and that have evaluative criteria based in a job
deliberately accomplished.

Previous work on math in daily contexts examines mathematics outside of school
settings. Research by Beach (1995), Lave (1988), Nunes et al. (1993), and others has shown
that success in various work and life practices depends on significant application of
mathematical concepts and principles. The research confirms that math is embedded in
people’s activities such as people on Weight Watchers’ diets solving fractions problems
(Lave 1988), Brazilian children developing strategies to assure profits in selling candy
(Saxe 1990), hospital nurses figuring drug doses (Hoyles et al. 2001), and high school
basketball players calculating their shooting percentages (Nasir 2000). One exciting con-
clusion from these studies is that reasoning mathematically in practical situations is wide-
spread in daily activity. A less hopeful finding is that, when given a school math problem
similar to those successfully solved in daily life, many people were not as successful.
Although these studies indicated that daily life was math rich, they did not address or
describe families in math engagement. Ginsberg et al. (1999) studied mathematical teach-
ing and learning and showed that the topics of number, ordinality, and the geometry of
shape were prevalent, and “pedagogy” varied across households. Rogoff et al. (1984)
discovered that parental instruction in math changed once children entered school. We
wanted to explore how family and math intersected in the middle school years when
mathematical foundations should be set.

We shared the general recognition that parents are the primary, most committed, and
effective educators of their children, and we shared the notion that parental involvement
may override factors such as family income or education (for classic statements, see
Coleman et al. 1966; Henderson and Berla 1994). Although educators are aware of the need
to involve parents, and a few success stories are inspiring (Epstein et al. 2002), most
schools have not found meaningful ways to involve parents, or involve them in ways that
counteract racial or socioeconomic inequities and lead to parent enfranchisement or
achievement gains (Casanova 1996; de Carvalho 2001; Fine 1993; Jackson and Remillard
2005).

Three projects that begin to reorganize school–family interactions around mathematics
and inspired our own project are worthy of note here. Family Math at the Lawrence Hall
of Science centered on the idea that family members are the first and most influential
teachers. They supported math workshops for families and the training of workshop
leaders, and set the stage by bringing parents together with school math and facilitating
math experiences to be done at home. The Funds of Knowledge and Bridges project at the
University of Arizona brought scholars, teacher educators, and teachers together as eth-
nographers to learn about the “funds of knowledge” families possessed, with the convic-
tion that they could be used as resources in schools (Gonzalez et al. 2005). A key feature of
this work is that it rejects a deprivation model of families and communities and brings
teachers into the critical work and discussions with families about what knowledge is,
how it is transacted within the community, and how it can support learning of school
subjects such as mathematics (Civil 2002). The effort to enfranchise families is also taken
up by the Algebra Project, which is both a math and a technological-age civil rights
movement, organizing families and communities to make algebra accessible to all middle
school students in the belief that it will enable enrollment in high school science and math
classes, opening gateways for later success (Moses and Cobb 2002). Our findings spotlight
the family as an accomplished site for mathematics work and recognize the potential for
it to be a more deliberate partner in school math success. Highlighting the mathematics
accomplished by families adds to our knowledge concerning how people solve mathe-
matical problems in various contexts, the increased role that families can play, and the
promise for involving them.
The Primes Project

We worked four years on a project, Primes, with the goal of increasing parents’ confidence with school math and participation with the school.1 We built our work on two hunches: (1) that parents engage in and utilize math in their households; and, (2) once invited to do so, parents could use their daily competencies as a base to support their children’s success in math at school. A goal of the project was to understand math at home and math at school, to help parents recognize possible connections, and to develop materials and activities that could help create linkages. Our goal was to identify everyday math accomplishments and show how they contained topics, skills, concepts, and representations of school math. Our hope was through increasing parents’ familiarity and confidence about the math they already did, we could design new ways to facilitate their engagements with their children’s school math. A second goal was to foster parents’ understanding of middle school math through creating advocacy materials that focused outside of the math itself, such as becoming knowledgeable about school math programs, homework and testing policies, events and support opportunities.

Our project comprised teams of parents, educators, community members, and middle school students from four Bay Area cities who joined our group, which included researchers, educators, curriculum developers, and math experts. We had previously developed middle school curriculum and demonstrated that real-world problem solving contexts gave many students better chances at school math learning and success (Middle-School Mathematics through Applications Project [MMAP] 1998); in Primes our task was to connect families to school math, and to do that, we needed to explore how family contexts might contribute in positive ways. Ultimately, the Primes group conducted research over a four year period and developed materials to help parents learn more about school math.2 The idea was that the parents and researchers in our group of 30 would learn from each other, and the designs we created would address the needs of other parents in their home communities.

The research aspects of our work were to help structure our discussions, reflect deeply, and make us responsive. Our perspective was deeply ethnographic, participatory, action-oriented, and critical in terms of how we—together—saw people’s mathematically relevant problem solving, school participation, and understanding of both. Our goal was to tend to the alignment of our emerging solutions for parents with the settings in which they were enacted (schools, community organizations, churches, television, and homes), and as understood by those who participated. The Primes group saw itself in an iterative process of inquiry and production, and we viewed our work at the intersection of ethnography and design-based research. Barab et al. (2004) called similar research “critical design ethnography.” Critical design ethnography starts with ethnography of a setting and an aim to transform the context while producing instructional designs with wider application. We characterize our research and design process as “hand-in-hand” because we started with a first step of mutual commitment to advocacy and then moved to a mutually constituting ethnographic and design-driven action. Reciprocity was implicit in the collaboration.

As a substudy, we worked with six families to develop observational case studies described herein, and include stories of three families in this article. We met two of the three families through our team members’ networks, and the third family members were long-term project partners. When we asked, they allowed us to visit and videotape them in their homes, workplaces, schools, and outings. The families came from three cities in the San Francisco Bay Area. They represented different points on three continuums of interest: (1) their socioeconomic situations; (2) the ways they interfaced with schools; and (3) their awareness of when and how daily problem solving was school relevant. Two of the families
demonstrated the challenges of supporting their children and their academic progress in the face of extremely tight financial constraints. In every case, the parents developed creative ways to handle work and income, to maintain connections with their children’s schools, and to keep abreast of their children’s academic progress.

Differences in the families in relation to math were both subtle and striking. In one family, only simple calculations and math facts were discussed with the children. A second family weaved school-like mathematical talk with everyday family experiences and suggested problems for the children to solve as they presented themselves in their activities. The third family sought ways to incorporate school math practices into their discussions. Each family tended to have a problem-solving comfort zone where they excelled and had a great deal of experience: one at budgeting and making ends meet; the next at planning for the future and keeping records, schedules and charts; the last at optimizing time in addressing day-to-day needs and schedules.

The process of understanding family math practices was dependent on our scouting family activities and events that had math potential, following along and observing, and reflecting and talking them through with the families. Although our interactions varied with each case study family, we were able to spend approximately 40 hours observing, interviewing, and videotaping each family over a period of several days to several weeks depending on families’ activities and schedules.

The first meeting in each home involved observing and videotaping family members as they moved through a day, and interviewing them about their interests, hobbies, family activities, and special events. We asked them about the mathematics they did. For the most part, the parents first identified the math they encountered at work and in home budgeting. Most family members said they used arithmetic regularly, and “arithmetic” was a label given to many activities involving work with numbers such as juggling complicated personal finances or completing home improvement projects. The children had much to report about school math, but all reported that school math, except for homework, was removed from their other life activities.

After initial interviews, observations, and discussions, we inventoried the activities that family members said they did individually and together. We were most interested in the “family” activities because they provided the most opportunities for shared math experiences. We sought to identify problems that were being solved, mathematics topics, reasoning, explanation, generalizations, and tool use. We tracked the activity contexts carefully, and our inventory revealed that some of the activities, such as building shelves, participating in sports, playing games, fundraising for charities and causes, sewing, dancing, planning and taking family trips, and cooking were rich contexts for mathematics. This process helped us see mathematics in each family’s activities. We then asked each to allow us to observe and videotape them from start to finish as they moved through their likely math-relevant activities.

We spent two to four days videotaping each family. For instance, when mom and daughter were sewing costumes for a dance recital, we filmed them for the sewing day, a rehearsal, and the performance day. Once we filmed, our research team analyzed the tapes and the analysis was iterative and ongoing. Content logs of all tapes were generated, identifying activities, events, and conversations, trying to establish their starts and finishes. Next we searched for mathematically involved activities, moments, or actions, and wherever possible, followed them from inception to their conclusions. Some events coded as math took seconds, but others, such as budgeting were periodic and nested in ongoing activity. In the case of ongoing events, we analyzed their constellations and varieties of engagement—from single instantiated moment (e.g., deciding if something is too expensive to purchase) to ongoing (e.g., making the mortgage payment and managing cash flow). If we could not see the start to finish, we asked the family members to fill in the
blanks. We invited the families to view the tapes and comment on them and our ideas about them. We called them back for more or qualifying information about their situations and problem solving. We had mathematics experts, teachers, and the parents looking together.

We sought to identify math in action and problems over time. We coded each math story arc first in open-coding mode, and eventually with nested coding categories. A sample of major categories included: activity context (i.e., with reference to all institutional connections, origin of activity), participants (present and essentially involved, alone or with others), ways of talking (specialized words, vocabulary, shorthands), math topics (i.e., arithmetic, algebra, measurement, logic, geometry, optimization), math processes (i.e., explanations, generalization, representational use, strategies); approaches (i.e., dive right in, ask for help, find a workaround, avoid math); tools (i.e., rulers, calculators, calendars, printed schedules, maps). Ultimately, we analyzed the intersection of the various perspectives we had on the activities and events and created a case study profile for each family to depict its math involvements based on five criteria: (1) the children were involved actively or passively in the “problems” (e.g., excluded work oriented problems that only involved parents); (2) families generated the problem in context (excluded working homework problems or problems posed by the research team); (3) the particular activity was one of many that involved math; (4) we could see the problem being solved in action; and (5) we could report on each family’s patterns of school involvements. All profiles were approved by the families.

Mathematics, Family Style

In the accounts that follow, we introduce each family and highlight the role of the context in generating the mathematics opportunities and examine mathematics problem solving that is both elegant and complex. The stories highlight the three main types of problem solving we saw across interviews and observations. Modeling was a practice many parents engaged in by talking through a problem solving process in the presence of their children. Prompting was an extension of this practice in which parents would ask their children to venture a solution. Distributed problem solving involved the family deliberately sharing the problem solving across members.

The Honey Family Prepares for the Prom

We met Pat Honey at the local photocopy center where she worked part time as a manager, processing our copying. Pat is an energetic African American woman who used to run a beauty shop. She is a single mom raising her two children and her niece. She found that the long hours trying to keep the shop afloat did not allow her to spend time with her family or to keep up with their academic needs. So, Pat made a decision to work part time at the copy center. To help make ends meet, she started a cash business selling hair extensions from her home. She balanced the family income around these two part-time jobs, because it allowed her to spend more time with her children, who ranged in age from middle school through high school. Pat’s hair business was completely integrated with family life. Everyone in the household worked in the hair business, helping Pat with sales and delivery of hair. They noted sales and inventory changes in a spreadsheet and used price charts, sales tax tables, and sales receipts.

Pat was active in the community, personally and professionally networking with people through her hair business and the copy center. In particular, Pat maintained ties with her children’s schools by brokering a relationship between the local copy center she managed and the local schools to provide supplies and service support. She had a keen eye for
community cohesiveness, and her philosophy about school involvement was built mainly on maintaining visibility and positive relationships with the school staff. For instance, Pat kept in touch with the principal of her daughter’s school, learned of the school’s needs, and enlisted the help of coworkers and the copy shop. In one sense, Pat was a model for parent involvement. In another, her relationships with the school did not move beyond the social into academically influential ones.

**Setting the Budget.** Money was tight, but Pat managed to save for special occasions. The prom was such an occasion. Star, Pat’s niece, wanted to attend the school prom, and prom preparation set the stage for the family members’ demonstrations of their abilities to primarily attend to family life while elegantly managing complex budget problem solving situations. When Pat sat down with her daughter, Milla, and Star to discuss the prom and set a budget, their conversation started with the dress and what Star would like, creating a learning opportunity involving money and budgets.

**Pat:** Star, I’m working with a tight budget, but I want you to go to the prom. I’m working with about two hundred dollars. It’s not much money, but I had to pay house taxes, had to pay house insurance and my regular bills. So, I did put aside some money and I do have at least two hundred dollars.

Once the total amount that could be spent on the prom was established, the family started to populate the budget. Pat had a pencil, paper, and a calculator at the table. She input amounts and recorded a list on the paper. Pat started the conversation by asking, “So, what do we need to buy?” Star answered, “shoes, stockings, nails.” Pat added a limo to the list, and Pat suggested that a limo was probably too expensive, but didn’t completely rule it out. The two began establishing their priorities: Pat wanted Star to dress appropriately for the prom and to do it on budget, while Star wanted to travel in style. By the end of the conversation they determined that they had $90.00 available for the prom dress and the limo was contingent on leftover money. The budget was a family matter, and in the conversation, Pat informed Milla that she was going to sacrifice some of her usual spending money for Star to attend the prom. Milla agreed reluctantly and then the three headed to an outlet store for dress shopping.

Although the budget appeared, at first, to be a straightforward case of addition and subtraction, it became more complex inside the social interaction of the family. In the space of a five-minute conversation, Pat, Star, and Milla engaged in a budget negotiation to optimize everyone’s needs. They took into account the fixed and negotiable areas of the budget, prioritizing expenses, and laid the groundwork for future negotiation.

**Buying the Dress.** Once the family started shopping for prom, Pat and Star were determined to stay within budget. As they looked through dress racks, Pat not only accounted for price but also figured discounts and taxes out loud before letting Star try them on, occasionally requesting the girls’ participation.

**Pat:** What’s the tax in this area Star?
**Star:** 8.25 percent.
**Pat:** Okay, this is 109, and 33 percent off. Like this one?
**Star:** No.
**Pat:** Okay, because that’s not too bad, it’s 30 percent off, 109, it’s about 78, 79 dollars, with tax, about 84, 85 dollars. You don’t like this one? [Star shakes her head, No.] Okay, let’s keep looking. This one’s 142, it’s a little too expensive.

At first Pat calculated the price of dresses even if Star was not interested in them. Subtly, she modeled two problem solving skills. By pushing the interaction a bit beyond Star’s
rejection of the dress, Pat demonstrated efficiency in shopping, and she set Star’s expectations regarding what was in the price range and what was too expensive. She also demonstrated the figuring of both the discount and the tax to ensure they didn’t end up surprised and run over budget at the register. They learned from the calculations that a dress marked $109 was in the range, but one marked $142 was not. We note the seamlessness of these calculations for Pat in contrast to what we generally know of math classrooms where this kind of multistep problem can raise challenges for students. According to Pat’s strategy, she figured the discount, subtracted it from the original price, figured the tax based on the discounted price, and added that amount to come up with the total cost. Pat used the calculation as a budget reminder and they continued to look.

Pat: Okay, let’s see. Green? What’s the price of this one? 109 dollars? That’s the same price. It’s about 84–85-dollars with the tax, with the 30 percent off. You like that one?

Star: Yea, but let’s keep looking.

Being involved with sales and tax at her jobs, Pat was used to doing quick mental calculations to estimate price—taking off the 33 percent discount then adding back the sales tax all in her head.

Milla: Star, how ’bout this one.

Star: Oh yea, Pat! Look at this one. (She holds it up in front of her).

Pat: Oh, and look at the price. Okay, that’s affordable Star, it’s on sale. How much is it, 89-dollars? With 30 percent off and the tax . . . it’s about 67 dollars for the dress Star. It’s affordable, and we’ll have a little extra left. We can put it on your pedicure or your manicure.

Star: Or the limo.

Pat’s estimations were within a dollar of the actual price. With the choice of an $89.00 dress, both Star and Pat were running budgets in their heads. Star figured on saving for a limo, while Pat figured on pedicures and manicures. The point is that the family members used adaptive strategies to talk as they shopped, and the mathematics work that Pat did was part of the conversational flow as they looked through racks of dresses. Pat was a quick calculator. She figured the price of the dress, the discounts and sales tax on the spot, and Star only tried on dresses within their budget (see Figure 1).

Prom Night Success. On prom night, Pat, Star, Star’s mom, and some neighboring families gathered as Star got ready. Pat and Star evaluated the success of their budgeting mission. Star had a dress, shoes, a manicure, a unique hairdo—and a limousine! Both had much to report as they tallied costs and management of the budget:

Pat: The dress, it didn’t end up being 90 dollars, the dress was 65. The shoes was 35, the stockings went over—they was 10 bucks, the nails instead of 20 were 15. So that’s how we made it. We made it though, so now we’re waiting on the limo, and it all worked itself out.

Star: I think I did pretty good for my first time, ’cause I picked out my dress like this, my mom picked out my shoes, my auntie bought me ten dollar stockings, and let me use her expensive pearls. I got my nails done, my cousin did my toes, and my auntie arranged for my hair to get done. I knew how much to spend and how much I couldn’t spend. I knew my range. So, if we went over it wasn’t my fault, it was her fault ’cause she kept wanting me to get more and more.

Youngster (outside): The limo’s here!!

Star made sure the limo fit into the plan. Her budget reflections revealed that she attended to the budget categories and amounts to which she had agreed, and that she understood how to make trade-offs in costs to maintain the bottom line. She also recognized that she accomplished something important in setting and meeting a budget. If going to the prom is considered one rite of passage, then managing the budget is another. Mental math,
number, and estimation came to the forefront of the family budget practices. Once Star and Pat figured the range and knew the amount of the dress and the cost of the prom ticket, they arrived at the discretionary dollar amount they had to play with as they juggled other expenses. With all of this negotiation and activity across family members and friends, Star learned to practice budgeting to get to the prom on her own terms while keeping the family content. Star recognized the importance of this accomplishment when she said, “I think I did pretty good for my first time.” What better way to begin to learn skills than to have them motivated by your social and familial desires and rites of passage? The mathematics skills involving calculation, estimation, prioritization, and mental mathematics became embedded and accomplished in the excitement of the prom and across the family members and their activities. All enjoyed the triumph, taking photos together and gathering outside to laugh and wave as Star pulled away in the limousine.

The prom and the hair business illustrate the Honey family’s efforts to create more time for each other and to stretch a limited budget to do special things. They are also among the many examples of the family tackling math-rich problems. Still, Pat and the kids rarely thought of themselves as transacting mathematically. Family life for the Honeys was so blended with problem solving that the family was present for, and engaged in, mathematical problem solving as a matter of course.

The Ances Family: Charting the Future

When we met the Ances family, they had recently purchased and moved into a new home. To afford exorbitant prices, they chose to live further outside the city and to make a long, daily commute that is so common for middle-income families in the Bay Area. Michael is an inquisitive and adventurous Filipino man who had many ideas for turning their home into their dream home. Karen, a white woman, matched her husband’s energy with a passion for sewing and dance, and designed costumes and clothes for the dance troupe she attended with her daughter. Michael and Karen both have professional degrees, both work, and share economic responsibility. Lauren was in eighth grade and Evan was in fifth grade.

![Figure 1. Managing the prom budget.](image-url)
The Ances family members were future oriented. It was a part of the family culture that was expressed time and again through elaborate savings plans, home improvement projects, and even the types of brain teasing problems they made up to solve together. Michael kept a “thermometer” chart of the money he was saving toward a car he wanted. When Karen and Lauren made costumes for the girls in Lauren’s dance troop recital, Karen kept a notebook with the fabric calculations, measurements, and sketches that she would also rely on for future projects. Daily, monthly, and yearly schedules were around the house. All of these ways of recording, remembering, keeping track of, and working with data made use of mathematical representations and logic schemes. All in all, the practice of charting information and keeping records was a zone of familiarity for the Ances family members.

The Ances family would get good marks from the school on the parental involvement front. Parents Michael and Karen had some knowledge of what was expected in the school curriculum and held their own in keeping up with the demands from the school for parental support. Their schedules allowed for them to attend parent nights, help their children with homework assignments, and proctor homework and test preparation. When they saw a math connection point, they used some of the representations, algorithms, symbols, and language to connect to the kids’ school math. They aimed deliberately toward college for their children. When we met and interviewed the Ances family they stood out as having a mathematics-rich array of family activities. Like many other families, they were quick to name calculations as the mathematics they did at home, yet did not readily recognize other types of mathematical activity they engaged. For instance, we observed Karen explaining a geometry rule about the equivalence of circles as she sewed costumes with Lauren, but she contended this was not really math, “just sewing.” We mention this because it was rich in mathematical thinking, yet the family engaged it as a matter of practical work without attention to the mathematics.

A Day at the Ballpark: The Warmup. The Ances family likes to make a day of baseball. They attended several professional games each year and Evan played on a Little League team. We accompanied the foursome to a game between the San Francisco Giants and the Chicago Cubs. They took newspapers to look up players’ stats and talked about records past and yet to be made. The family’s day at the ballpark had several notable features. The familial context for the math—the baseball game—was extremely conducive to math problem creation and problem solving. Baseball is a game to watch, but is made richer by a discourse based on records and statistics. To talk baseball is to transact in game, season, and lifetime averages and percentages. When the Ances family watched the game, they chatted, snacked, talked baseball, and in the process, used math consciously, especially statistics to problem solve around topics that entertained them. Their ballpark interactions instantiated math in language, use of symbols, processes, and algorithms. Early in the game, Michael and Evan discussed game hits and batting averages.

Karen: Okay, here comes Bernard.
Evan: Bernard. What did he hit?
Michael: He’s got a little bit better average, he’s at .277.

While the family waited for game action, Michael asked Evan about his Little League statistics, drawing links between Evan’s stats and those of professional players. Evan’s stats were kept in a notebook he had with him, and he pointed to each column and announced what they represented (see Figure 2).

Michael: You want to take a look at your stats? Can you tell me what all these were?
Evan: Okay, that’s hit by pitch.
Michael: I’m glad there weren’t a lot of those!
Evan: One. At bats, there were 6, 9, 11.
Michael: Oh, you didn’t total it?
Evan: Yea. That’s hits, doubles, triples, home runs, base on balls, which is also known as walks, strike outs, average for the game, average for the season, runs I scored, RBIs, and stolen bases.

Charting baseball stats was one way Evan was both becoming a legitimate player and fan of baseball, a process Michael supported during this game. It also practiced Evan on math skills through creating and using charts for record keeping and interpreting his progress. Michael focused on the statistics, asking Evan which was his best batting average. Evan’s answer of a .666 average in two games prompted a deeper look.

Michael: How’d you get that?
Evan: You do your hits, divided by your at bats.

Evan used a simple formula to turn his raw stats into a batting average that told him how he did in a single game—or throughout a full season. His data handling indicated that he knew it wasn’t just about numbers, but about which numbers were important in particular situations. When Michael asked which “at bats” were counted in the batting average figures, Evan answered:

Evan: Yea, you don’t count the walks, uhm. . . .
Michael: You probably don’t count the hit by a pitch. The reason why is, like if you get someone like Barry Bonds, they’ll try to walk him all the time . . . and his average shouldn’t go down just because they keep walking him.

Michael integrated Evan’s stats with an explanation of a similar situation for professional slugger, Barry Bonds. They talked about stats when players came up to bat, when there was a hit or an error, and when there was a paucity of game action. They covered Evan’s progress, how the pro players were doing, and how the teams were comparing. Topics were generated in the moment and often in relation to what happened in the game.
Information they needed to sustain the talk was available in the journal, the newspaper, the scoreboard, and in their memories.

Michael initiated a conversation comparing a player’s performance from one year to the next when Sammy Sosa of the Chicago Cubs came up to bat, suggesting that they check to see if Sosa was hitting home runs this year at a pace that matched the year he broke Babe Ruth’s record of 60. He then started seeking out the factors that would help them figure the answer. Smoothly, and with attention to the game context and the whole family’s participation, Michael set the stage for solving a hypothetical and forward projection problem. Evan took up this challenge, supplying some information from the newspaper.

Michael: So, let’s figure out if Sosa’s on pace. So how many games were there now?
Evan: 64.
Michael: He’s got how many home runs?
Evan:—(gets help from Michael to look it up in the newspaper)—23.

Michael and Evan assembled this information. Michael already knew the number of regular season games—162. Once they gathered the information Evan suggested that Lauren solve the problem. Without hesitation, he passed her the information and the notebook. Handing the problem to Lauren at the point where it required a more difficult level of math indicated an awareness across the family of differential skills and knowledge.

Michael: Lauren knows how to do that.
Evan: Sosa has 23 home runs in 64 games, and there are 162 games in a season,
Lauren: Ok. (Lauren starts writing down information.)
Evan: So, how many home runs will he have . . . ?
Lauren: Ok.
Karen: Oh, if he keeps going at this rate?
Evan: Yea.

Lauren set up the problem using the notebook and pencil while Karen looked on.

Lauren: Well you have to do 23 home runs over 64 games and then $n$, your unknown is over 162 cause that’s the total, right? (Lauren works out the problem on paper.)
Lauren: So it’s about 58.
Karen: 58 runs? And how many did he have last year Evan?
Evan: 66.
Karen: 66 runs last year. Ahhhh, he’s slower, he’s gonna have to pick it up if he wants to hit 66 again.

Immediately following the solution announcement the crowd roared and the family returned to the game. Together, they defined and figured a multilevel problem that engaged them in the baseball game context. They also did distributed problem solving, with Michael constructing the problem, Evan identifying the necessary constants and handing the problem off to Lauren, guessing she had the know-how he lacked for solving it. Lauren set up and solved for it, and Karen wrapped it up, clarifying where Sosa’s performance stood (see Figure 3).

Lauren’s solution method was typical for how this problem would be constructed and solved in a school math class. She set up a proportion problem and figured precisely, then rounded her answer to 58 because homeruns come in whole numbers.

To close, the Ances family was able to exercise mathematically in the context of leisure activities, defining context-relevant problems and delegating solution responsibilities to those who might be ready to take them on. They exhibited overlap between home and school math, yet remind us that the connections are not necessarily recognized or made.
Mae and Clark Delancey, African American parents living in Bay Area public housing, had two things on their minds—staying involved with their children and wanting them to know the importance of school. Mae worked nights as a nurse and found it difficult to see her children. To address this concern, Mae spent many days volunteering at the schools her three children attended as a way to stay involved with her children and to be in a position to advocate for them at school. She was in her daughter Danielle’s high school so regularly, they provided her a work area. The Delanceys were active in PTA and site council, parent advocacy groups, and community education groups. They wanted to help other parents and give their children the message that education was important and essential. Mae explains:

I didn’t see my kids for a while . . . cause I’m a nurse. I work at Kaiser Medical Center and at UC, and a lot of times I come home from work at six o’clock in the morning. They’re going to school, but I’m going to bed. And vice-versa. You know, so I took the time out to come up here [the school] and be with them. This is how I spend my time with them . . . they don’t like it but I do.

The Delanceys were members of the Primes team. Over the four years we observed a transition to more intentionality in the way Mae and Clark engaged their children with math. They gave much attention to their own problem-solving specialty—optimization within the bounds of numerous constraints. Transportation challenges permeated the daily life of this family and were a good example of this optimization work. The family did not own a car, and all family members spent a few hours a day on city buses. We traveled Mae’s routes with her and were astonished at how much information she gathered, organized, and put into play to choose bus routes. When a mathematician suggested the mathematical complexity of Mae’s routing accomplishments, she did not see math, only common sense and necessity. Routing and organization tasks were prominent when the family grocery shopped. They knew the bus route for the day and time, how many hands

---

### Figure 3.
Is Sosa on pace for a record number of homeruns this season?

<table>
<thead>
<tr>
<th>Setting up the ratio:</th>
<th>23</th>
<th>=</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>162</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solving for N using cross multiplication:

\[
\frac{23}{64} = \frac{N}{162}
\]

\[64N = 23 \times 162\]

\[64N = 3726\]

\[N = \frac{3726}{64}\]

\[N = 58.22 \text{ or about } 58 \text{ homeruns}\]

Solution: At this pace, Sosa will hit fewer homeruns than in his record-setting year.

---

When the problem gets to Lauren, she has the necessary information to solve it:

<table>
<thead>
<tr>
<th># of games so far this season:</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td># of homeruns hit this season:</td>
<td>23</td>
</tr>
</tbody>
</table>

### The Delancey Family

Mae and Clark Delancey, African American parents living in Bay Area public housing, had two things on their minds—staying involved with their children and wanting them to know the importance of school. Mae worked nights as a nurse and found it difficult to see her children. To address this concern, Mae spent many days volunteering at the schools her three children attended as a way to stay involved with her children and to be in a position to advocate for them at school. She was in her daughter Danielle’s high school so regularly, they provided her a work area. The Delanceys were active in PTA and site council, parent advocacy groups, and community education groups. They wanted to help other parents and give their children the message that education was important and essential. Mae explains:

I didn’t see my kids for a while . . . cause I’m a nurse. I work at Kaiser Medical Center and at UC, and a lot of times I come home from work at six o’clock in the morning. They’re going to school, but I’m going to bed. And vice-versa. You know, so I took the time out to come up here [the school] and be with them. This is how I spend my time with them . . . they don’t like it but I do.

The Delanceys were members of the Primes team. Over the four years we observed a transition to more intentionality in the way Mae and Clark engaged their children with math. They gave much attention to their own problem-solving specialty—optimization within the bounds of numerous constraints. Transportation challenges permeated the daily life of this family and were a good example of this optimization work. The family did not own a car, and all family members spent a few hours a day on city buses. We traveled Mae’s routes with her and were astonished at how much information she gathered, organized, and put into play to choose bus routes. When a mathematician suggested the mathematical complexity of Mae’s routing accomplishments, she did not see math, only common sense and necessity. Routing and organization tasks were prominent when the family grocery shopped. They knew the bus route for the day and time, how many hands
they needed to carry the week’s groceries, and most importantly, their budget. Shopping with the Delanceys helped us learn why families see common sense where our trained eyes saw math. Comparison shopping and decision making took place regularly, and over time had become routinized. The whole family could quickly distinguish between times when price comparisons were required and times the answers were simply known. Their weekly shopping was completed in just an hour and a half, all within budget, and coordinating the diverging schedules of three teens and two adults. This required activities including data gathering, understanding variables and proportions, interacting with representations of information, and solving rate, time, and distance problems.

Getting to School on Time: Juggling Constraints. Mae and Clark transitioned from solving daily life problems for the children to solving them with the children. They were aware of this change in their approach and invited us to look in on a family discussion to help figure out the best bus route for their twin boys to take to their new school. Their plan was to engage the whole family in a math-infused discussion. They gathered the family around a map on their coffee table, noticing that there were two buses (#19 and #44) that connected with the bus that runs by the school (#71). Clark wanted to find the fastest route, while son Derrick was willing to take any bus that arrived first at the closest bus stop. Mae was wary about connections in neighborhoods with gang activity, and she checked the routes intently. Clark insisted on gathering information and making an “informed” decision. He laid out a city map, a bus map, a schedule, and asked for a ruler. He then got everyone oriented to the map and used the scale to calculate the distance from home to school.

Clark: Okay, I got a map of [the city] and we’re gonna see exactly what this is all about. And, there’s also a transit map that should show you some of the time schedules. Okay, you guys help me locate it, now where’s [Garden] Point?
Danielle: [Garden] Point is right here.
Clark: All right. We’re gonna need a ruler. Erick, will you go get me a ruler? There’s a legend on the map that shows you what the distance is in miles. This line here is equal to one mile on this map. Every time you have this length, it’s one mile and according to this one, two-and-a-half inches equals one mile. How many times does 5 go into 18?
Derrick: About three.

Clark pointed out the scale bar on the map’s legend, and with the ruler, figured that two and a half inches on the map equaled one mile in the world. Doubling that, he reasoned that five inches on the map is two miles in the real world. He then measured the route lengths, and the others applied the scale to find the distance for each. To figure each route’s distance, they divided the map distance for each route by five, knowing that five inches on the map equaled two miles.

Clark: So that’s three fives, so each five is two miles, so that’s six miles and... it’s a little over seven miles, it’s almost seven and a half. Now which one was shorter? The first one was, six and four, is ten miles. The #19 is ten miles.

The number #19 bus trip was about 10 miles, and the #44 bus trip was seven and a half. Using the scale helped figure the shortest route, but that information wasn’t all that they considered (see Figure 4).

Experience told them that the shortest route wasn’t always the fastest. Other variables were considered. Was the bus an express? Did passing through downtown mean commuter traffic?

Mae: If he leaves here, say 7:30 or quarter to 8:00, what time is the bus going to be out there for him to actually get the bus and go through traffic and actually be there on time before the school bell rings?
Danielle: The 44 to 71 route takes about an hour and, I would say, 15 minutes. So, I’m not sure about the 19. The 19 may take longer.

Clark: But you’ve got to remember, it depends on what time of the day you’re going. If it’s early in the morning, the 19 is going to be crowded with people going to work. And if it’s early in the morning, the 44 is going to be crowded with school children. So the 19 would be your best bet because it’s inbound instead of outbound.

Danielle suggested that the #44 route was not only shorter but also it avoided the downtown traffic the #19 bus would encounter. Clark considered and figured. Danielle mentioned she had traveled the routes repeatedly and estimated each trip’s time. With this information, the family figured out how fast each bus traveled by dividing the length of the trip in minutes by the distance traveled. Rate, time, and distance are interrelated. If you know the values of any two, you can find the third. This rate, distance, and time problem—a simple linear equation—is prominent in middle school.

Clark: So, if you take the 19 route, it takes you 90 minutes and it takes you nine minutes a mile. It’s ten miles, so ten into 90 goes nine times so it takes you nine minutes a mile. If you take the 44 route, it takes you 45 minutes and it’s seven and a half miles. When you divide seven and a half into 45 you come out with six minutes per mile. So, you cut your time in half with this route, with the 44 route. It’s half the time of the 19 route. Overall distance, the 44 is shorter. So, I was wrong. The 19 is longer, you were right.

The Delanceys know that the #19 bus route takes about 90 minutes. That’s the time. They also know that the route is about ten miles long. That’s the distance. They found the third value—the rate—by dividing the time by the distance, or 90 by ten, to get the rate of the #19 bus: nine minutes per mile (see Figure 5).

They use the same process to find that the #44 route would take about six minutes per mile. The figuring confirms that the #44 bus covers less distance and travels faster than the #19, making it the winning choice for Erick and Derrick.

The Delanceys went the distance to figure out the best way for the boys to get to their new school, and this was both an opportunity for solving a transportation problem and for making the mathematics obvious. Clark “performed out loud” the math he used, and his “math lesson” was significant. This is a classic school math problem as well as a common problem on standardized tests. The context of getting around San Francisco in the real world makes the problem more compelling and more complex than the school version.
Every time the family traveled they faced these choices: How often does the line run? What time of day will it be? Is the bus express? How close does it get me with the fewest connections? Weight assigned to these factors helped determine the routes. Clark and Mae had always solved these problems for their family. Through our collaborations they became aware of the math in their daily life activities and they saw a benefit to solving these problems in conversation with their children, distributing some decision-making practices in the process. It was significant that they began to take advantage of those opportunities intentionally.

Findings and Discussion

Families Do Math and Context Drives It

Families do a great deal of problem solving and troubleshooting as a matter of course, and quite a bit of family activity is mathematics relevant. Family members set goals, plan, create budgets, and do forecasting and reconciling of budgets and expenses. They make decisions after weighing all of the variables, dabble in statistics, and optimize. They think conceptually and logically, demonstrate flexibility and adaptability, search for patterns and discrepancies, develop strategies, use approximation, estimation, and make decisions based on priorities, multiple conditions, and variables.

The contexts families engage in define their needs to problem solve and also make accessible and ready the resources needed for getting to solutions. We saw families use calculators, rulers, pencil and paper, maps, newspapers, material goods, and even an occasional school-taught algorithm or formula. They used or designed representations of information and data such as charts, scoreboards, records, dynamic spreadsheets, calendars, and sketches. They used rounding, estimating, and approximating where they had leeway. They organized, categorized, and prioritized information and routinized their problem solving. We saw these strategies and engagements from family-to-family,
regardless of other, apparent differences. Whether they were college educated, life educated, or both, and regardless of race or ethnicity, mathematically relevant transactions abounded in the basics of their lives. The family members problem solved as needed, with each other, and in relation to the process of accomplishing life both inside and outside the household. The business of life, the activity contexts, who was present or needed, and available tools and resources comprised mathematical problem solving.

Families Don’t Call What They Do Math

Two additional findings are of interest. The first is that family members frequently did not see what they did as mathematics. They viewed their attentions and engagements as dealing with life; their definitions of the situations were context and activity based, not epistemological in nature. They did not think, “Do I solve this problem with math, science or through aesthetics?” They thought, “I’ve got this problem. I’ve done this before. Who or what around here tells me what to do or helps me?” Being in the store with dresses in hand, budget in mind, and prom to attend helps to organize the calculations. Still, the parents in our study did not typically think of their lives as mathematically rich or complex because they had not really considered the mathematics they practiced in their daily problem posing and problem solving beyond the more obvious basic calculations.

Life Practice and School Math Compose Family Math

The mathematics work we saw—arithmetic, categorization, figuring percentages, solving for $n$, ratio and proportions, linear algebra, use of multiple data representations, scale, rate problems, mental calculation, optimization, and rounding and estimation—are all identified in the national mathematics standards, and they are especially relevant to the standards up to and through the middle school level (NCTM 2000). We saw the use of appropriate methods and tools from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and an ability to judge the reasonableness of the results (NCTM 2000). The family members did rely on school-taught algorithms and strategies although they used few formalisms. Pat Honey figured percentages and discounts through mental calculations. Lauren Ances used a typical school method for solving the baseball problem. Clark Delancey used a standard rate formula for figuring out the school bus route. Yet they only occasionally wrote or stated a formula before they leapt into problem-solving action. They combined their prior practices, routines, and the resources at hand as heuristics for both problem definition and problem solution. Their math-related practices were combinations of what is covered in school and what is practiced in life, and they revealed that school math gets carried across from school to the daily life setting. We find that adults and kids are using some of the algorithms, skills, and formulas they learned in school such as the rate–time–distance formula and the process of setting up a proportional reasoning problem by solving for $X$. Finally, these combined approaches consistently led to effective mathematical problem solving.

These three findings—families do math regularly, don’t think of it as such, and that family problem solving is a combination of life practice and school math forms—lead us to reframe our initial question of, “are there connections to middle school math?” We ask, instead, “how might we help cultivate and legitimate the family as a site for math teaching and learning?” If we can view families as engaging math teaching and learning, we can ask a new question—“What does this buy us in terms of understanding family involvement as a function of kids’ learning in general and math learning success specifically?”
We conclude that we need to return to work with families to push a “math in the family agenda” even further. We agree with the plea by McDermott and Webber (1998:323) to provide more opportunities for math moments “to overlap systematically with the lives of children.” We call more stakeholders to the table as we try to achieve successful math learning for our children. We have to invite families into the discourse that will allow them to share, discover, and discuss their math relevant situations. In our related Primes project work we learned that families can see the math in their problem solving when they have supports for doing so. Experiences that help parents redefine the definitions of their life situations to include math can be helpful (Steen 2004). To redefine, it helps to show how a life problem’s solutions are similar to those in classroom or test problems. We did some of this work in the Primes project and many parents reported a boost in their confidence. It made people want to involve the family in problem solving with math.

The Honey, Ances, and Delancey family cases suggest that much is possible when families are cast as legitimate, ongoing sites and agents for mathematics education. Parents, families, and communities provide rich and varied contexts for mathematical work, and acknowledging this reality can open up the ways parents might interact with school mathematics. Educators have expected and relied on parents helping their children with school math, and parents seem to take on the responsibility both consciously and unconsciously through modeling and distributing problem-solving practices among family members. When the family is recognized as a unique and legitimate cultural site for math learning, by parents and educators alike, two needs can be more effectively met. First, the traditional expectation that parents will help their children with school math is more likely to be met as parents’ confidence increases and as the school promotes respect for the family’s unique mathematical contexts. Second, the rich contexts for mathematical work, created and supported in the daily life of families can be intentionally put to work by parents in support of their children’s learning. The Delancey case revealed that interactions that highlighted and respected math practice in family life could open up a new realm for parental support of their children’s learning. Identifying the presence of mathematical problem solving opportunities in family life was the first step. More in depth interactions over time led to attempts to move beyond identification of math practice to intentional problem design for children to get more direct practice with problem posing and problem solving. In effect, parents discovered a new store of academic capital and began putting it to work.

When the school and the family are both recognized as sites for math learning, opportunities to engage math as a vibrant part of daily life and learning multiply. This need not be a matter of increased burden on schools to engage families or vice versa. It is a matter of respecting and acknowledging the legitimacy of math practice in and outside of school. If educators have access to the most common forms of context-based mathematical practice in families (Goldman et al. 2006), this could provide a reference for future school and family partnerships.

Notes

Acknowledgments. We thank the families who let us into their lives and became our teachers. Without them, this research would not have been possible. Perry Gilmore and Ray McDermott read the manuscript in ways that helped us develop it, as did the AEQ reviewers.

1. Primes was supported through a grant from the National Science Foundation. The research and findings are the responsibility of the authors and not the opinion of the Foundation.

2. We created nine workshops for parents that built off daily life familial contexts to give them an experience with middle-grades mathematics. The group also developed a guide called *Middle School*
Mathematics: What Every Parent Should Know and Can Do to help parents negotiate school math topics and processes. A television special based on this research called The Family Angle was created and broadcast by PBS channel KTEH in San Jose, CA.

3. Being followed by our team increased the family’s scheduling demands exponentially, but the families were patient. Our requirement of obtaining written approvals for videotaping and use of footage from locations the families frequented caused schedule problems and disruptions.

4. All family names are pseudonyms.

5. Organizing, prioritizing, and ranking are skills identified as defining mathematical work as well as more general life skills (Steen 2004).

6. Charting and making meaning of information and data are taught in elementary and middle school math classrooms. The national standards Data Analysis and Probability strand has fifth-grade students creating, organizing, and using data to problem solve with a variety of representations such as tables, line plots, bar graphs, and line graphs (NCTM 2000).

7. We saw this demonstration of understanding consistently across families in an evaluation study (Tucker 2002).

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